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RESEARCH ARTICLE

Mathematical characterisation of Bridget Riley's stripe paintings

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I investigate whether mathematical measures can characterise Bridget Riley's stripe paintings. This is motivated by three considerations: (1) stripe paintings are an incredibly constrained art form, therefore it should be relatively straightforward to ascertain whether or not there is a mathematical characterisation; (2) Bridget Riley's approach to composition is methodical and thoughtful, so we can assume that her paintings are carefully constructed rather than random; and (3) Riley's paintings can appear random on a first glance but have an underlying structure, therefore Riley's works are challenging to characterise because they are close to random while not actually being so. I investigate entropy (both global and local), separation distance, and auto-correlation. I find that all can provide some characterisation, that entropy provides the best judge between Riley's work and randomly-generated variants, and that the entropy measures correlate well with the art-critical descriptions of Riley's development of this style over the five years in which she worked with it.

Keywords: Bridget Riley, aesthetics, entropy, auto-correlation

AMS Subject Classification: 00A66, 62P99

1. Introduction

Many abstract artists have composed paintings comprising only vertical coloured stripes (e.g. Figure 1). Gene Davis (1920-1985) is the key proponent of this idea, using regular stripes in many ways from the late 1950s onward. Bridget Riley (b.1931) investigated them over five years in the early 1980s.

Regular stripes provide a compositional framework that has no context, that is, there is no analogue to anything in the natural world. It also provides a strong interaction between adjacent colours, that is, it provides the entire vertical edge between two stripes as an interface between their two colour fields. Regular stripes are an extremely constrained form of art, free of the contextual and cultural baggage that permeate other artworks. They are thus an interesting case study of the mathematical analysis of art.

I concentrate on Bridget Riley's work. This is because of Riley's approach to her art. I recognise that Riley is not a mathematician and says that she does not use mathematics in her work but she *is* extremely methodical and, as she puts it, has a "workmanlike" approach to her art [10]. She investigated the vertical stripe over five years (1980–85), and she put considerable effort into designing and developing her stripe paintings. In her work we have paintings that are, in some sense, optimum compositions. From observation of her working method, and reading of the commentaries on the work, we can reasonably assume that her finished paintings represent, for her, a carefully constructed optimisation of the sequence of colours. If our mathematical

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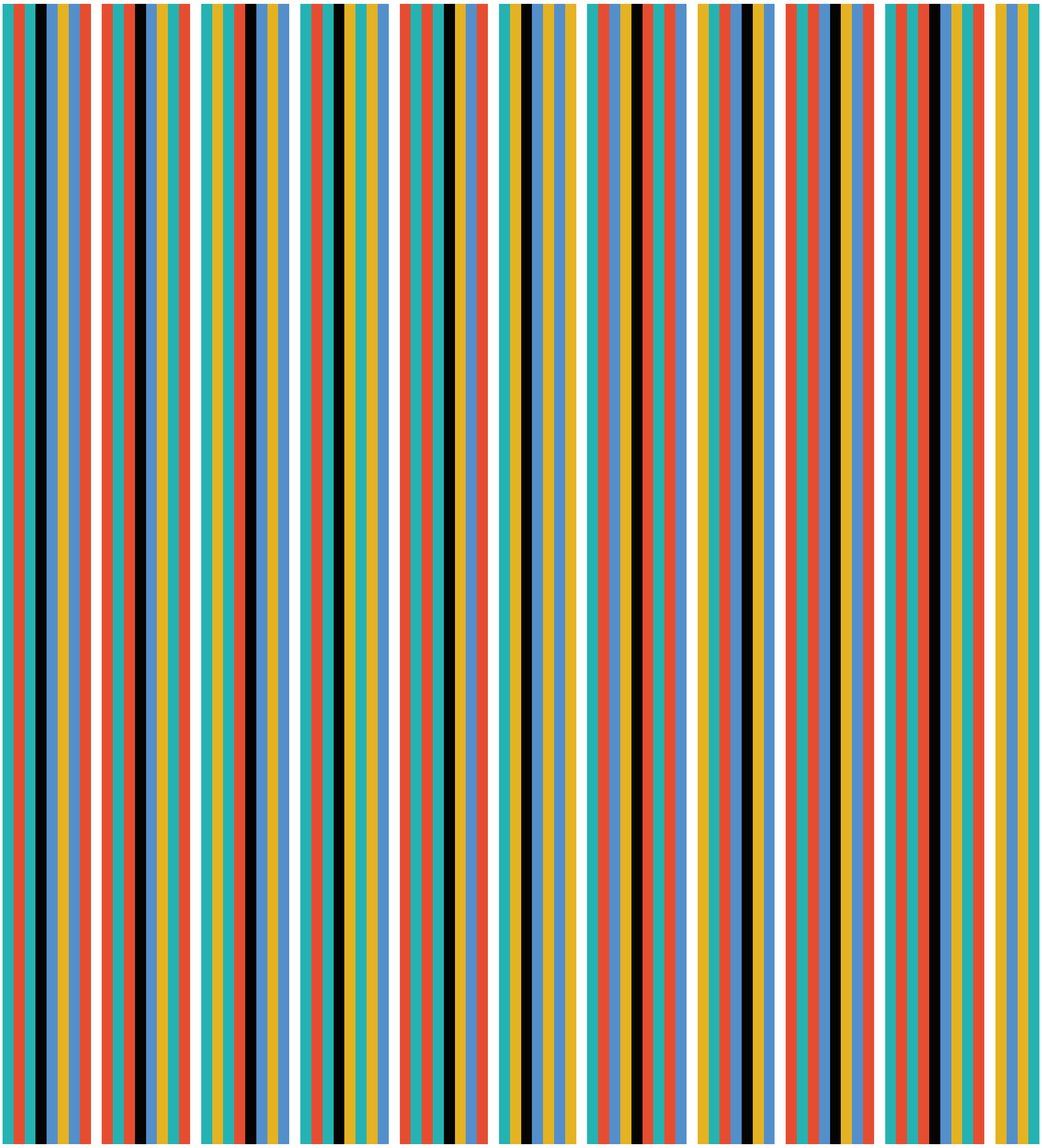


Figure 1. *Not Riley* (2011): an example of a stripe painting. This is generated using some of the features of Bridget Riley's stripe paintings, but with a slightly higher frequency of black and white lines. Ideally should be viewed at size 2 m × 2 m. ©2011 Neil Dodgson, reproduced with permission.

analysis is good, we should be able to characterise the mathematical features that comprise this optimisation and compare our analysis to what Riley says about these compositions. There is debate about whether such analysis is appropriate. Nettheim comments, with regard to the analysis of music, that "A work of art is individual and, although generally related to a tradition, to some extent establishes its own terms of reference, rather than being a replication or the output of a production line." [18] Riley's stripe paintings are an interesting test case because, while we do not have a production line, we do have several exemplars, rather than just one, and we have art-critical documentation of how the style developed between the exemplars [12].

I should be clear from the start that I am not going to Riley to support some pet theory about art, rather I am using her carefully constrained compositional framework to investigate whether it is possible to create any theory at all. One should also note that Riley does not have some fixed optimal composition: her compositions change throughout the early 1980s, as she developed this style, and so I would expect any analysis to be able to characterise that change also.

Riley's work contrasts with the earlier work of Gene Davis. Riley's compositions are carefully constructed, while Davis often worked on impulse. In particular Davis once spoke of painting stripes with acrylic on un-sized canvas as "a kind of artistic Russian roulette", since he could not "change the colour of a stripe once it [was] painted"; consequently, "each decision [had to] be the correct one." [13]. We should therefore expect it to be rather more difficult to find mathematical structure in Davis' spur-of-the-moment works than in Riley's methodically constructed works. Davis also produced much more structured work (e.g., Figure 2(i)), demonstrating that the humble stripe can be used to produce a painting that contains far more obvious structure than Riley's compositions, but which can still produce a strong response in the viewer. Riley's stripe paintings fall at an interesting place in compositional space, between the obviously regular and the purely random. Their structure is not immediately obvious to the naïve viewer. This makes them interesting to investigate as we would hope that mathematical analysis could draw out the subtle rhythms and patterns used by the artist.

I investigate three mathematical tools: entropy (both global and local), separation distance, and auto-correlation. These tools produce mathematical measures that allow us to characterise *some* of what Riley was doing and that correlate well with critical descriptions of her work. In particular, we can separate (most of) her compositions from randomly-generated compositions using entropy space. However, none of the tools produces a measure that reliably *identifies* Riley's paintings.

2. Background and context

Many abstract painters have used vertical stripes at some point. For example, in the New York Museum of Modern Art's seminal 1965 exhibition, *The Responsive Eye*, compositions comprising only vertical stripes of colour were exhibited by Gene Davis, Morris Louis and Guido Molinari [20][1]. Gene Davis remains the principal proponent of the vertical stripe [13]. Although he painted in a variety of styles over 35 years, many observers thought of him solely as the "stripe painter" [22]. The vertical stripe, as he said himself, is his basic subject matter. Davis acknowledged the "banal convention of the field of stripes" but he revolutionised it by complicating it. Some critics argued that his earlier stripe paintings were not art at all and belonged instead on wallpaper, curtains, and upholstery. Indeed, it is easy to find examples of stripe patterns in use on all of these everyday objects. One of the things that interests me is what differentiates the artwork from these mundane uses.

In 1987, Kuspit asserted that "there is still no satisfactory comprehension of Davis' stripe paintings. He worked 'without logic and without sequence' in order to be free to explore the seemingly inexhaustible possibilities of a particular stripe format" [13]. One could ask what it is that Davis, and later Riley, saw in painting vertical stripes. Davis says "a colour achieves its identity only through juxtaposition" [13]. Riley says "all colours vary according to their context and this 'relatedness' is in no other way accessible than through the particular change it effects" [10] The format of evenly sized vertical stripes allows the colours to interact with each other in a pure way. There is nothing else for human perception to latch onto than the colours, their sequence, and the way they interact with one another. Stripe painting fits with Riley's entire method of working. All of Riley's abstract art consists of working in one or other of her incredibly-constrained artistic domains, and then fully investigating what she can do within that domain. Boden [2] describes three forms of creativity: *combinational*, *exploratory*,

[1]Gene Davis: *Black-Grey Beat*, 1962, 62 even stripes; Morris Louis: *Number 33*, 1962, 19 irregular ragged stripes; Guido Molinari: *Mutation: Verte et Rouge*, 1964, eight even stripes.

and *transformational*; Riley's method is *exploratory*. It is eloquently expressed by Igor Stravinsky, in what Kudielka [12] calls Riley's "artistic credo":

> *My freedom thus consists in my moving about within the narrow frame that I have assigned myself for each one of my undertakings. I shall go even further: my freedom will be so much the greater and more meaningful, the more narrowly I limit my field of action and the more I surround myself with obstacles. Whatever diminishes constraint, diminishes strength. The more constraints one imposes, the more one frees one's self of the chains that shackle the spirit.* [24]

Riley was also an exhibitor at *The Responsive Eye* in 1965, but she did not come to paint evenly sized vertical stripes until 1980. We cannot know whether Davis' work from 15 years earlier influenced her, though she would obviously have been aware of it. As mentioned above, the two artists' approaches to composition were rather different: Riley adopting a careful, methodical development of each painting, where Davis is alleged to have been more impulsive. Nevertheless, some of Davis' compositions do have strong structure that belies this reputation for impulsiveness (e.g., *Black Grey Beat* (1964) [20], *Red Devil* (1959) see Fig. 2(i), *Solar Skin* (1964)).

Copyright considerations constrain what can be reproduced in this paper. Figure 1 is an example of a stripe painting, which I created for this paper in order to illustrate the genre. While we cannot reproduce other artists' works themselves, it is possible to show the colour sequences used: ten examples are given in Figure 2. An important thing to keep in mind is that stripe paintings need to be seen at their proper scale in order to appreciate the sensation that the artist desired. Bridget Riley is consistent in producing stripes about 19 mm wide on canvases about two meters square. Her stripe paintings usually comprise around 100 stripes. Gene Davis used a wider range of stripe widths and of numbers of stripes (e.g., the three paintings listed at the end of the previous paragraphs have parameters 62 stripes of 75 mm, 65 stripes of 28 mm, and 32 stripes of 75 mm). Davis' paintings are generally also about two metres tall. There seems to be an artistic or psychological need to have the stripes at least the same height as the human viewer. Abstract art tends to be large, which may be related to the concept of the *sublime*: that is, something that causes awe as opposed to something that is considered beautiful [17]. "Greatness of dimension is a powerful cause of the sublime." [4]

Riley's early paintings are well-known for their disturbing optical effects. These stripe paintings, by contrast, should not induce discomfort when viewed at their proper size. It is known that simple patterns of stripes with a spatial frequency within two octaves of three cycles per degree (i.e., 0.75–12 cycles per degree) can, in those who are susceptible, induce seizures and headaches, and will induce discomfort in normal individuals, and that subjective ratings of aversion correlate well with frequency within one octave of three cycles per degree (i.e., 1.5–6 cycles per degree) [6]. Some of Riley's well-known work from the Sixties does lie in this "discomfort zone." However, the stripe paintings from the Eighties have a spatial frequency of only half a cycle per degree, when viewed from one metre, so they lie outside this zone and their principal visual effect is caused by the interaction of the colours not the frequency of the stripes.

Bridget Riley developed her stripe paintings over five years, 1980 to 1985. There is a well-documented progression of her style over these years [12]. Simple observation makes it clear that her colour palette changed over this time. A good mathematical characterisation should be able to determine whether there is any mathematical difference in the other properties, such as the sequencing of colours.

Riley's stripe paintings have four developmental stages, documented by Kudielka [12]. In the first stage, black bands articulate the colour orchestration (e.g., *Summer's Field*, *Luxor*, *Serenissima*, *Rose Rime*, all 1982). In the second stage, this structural support of black is dispensed with (e.g., *Bali*, *Greensleeves*, *Cherry Autumn*, all 1983), next lilac is substituted for white (e.g., *Tabriz*, *Coxcomb*, both 1984), and finally the limits of the palette are overturned by a burst of colour (e.g., *Saraband*, *Burnished Sky*, both 1985).

The first three stages used just six colours in a painting. The first stage using Riley's *Egyptian palette* of black, white, blue, turquoise, red, and yellow. The fourth stage moves to a larger palette. For example, *Saraband* (1985) [19] contains at least ten colours (two pinks, three greens,

(a) *Silvered* (1981), 109 stripes, 25 turquoise, 21 red, 21 blue, 25 yellow, 9 black, 8 white

(b) *Après Midi* (1981), 105 stripes, 32 turquoise, 13 red, 22 blue, 18 yellow, 12 black, 8 white

(c) *Summer's Field* (1982), 78 stripes, 21 turquoise, 9 red, 17 blue, 18 yellow, 7 black, 6 white

(d) *Serenissima* (1982), 103 stripes, 21 turquoise, 22 red, 21 blue, 23 yellow, 7 black, 9 white

(e) *Luxor* (1982), 103 stripes, 29 turquoise, 14 red, 26 blue, 18 yellow, 10 black, 6 white

(f) *Bali* (1983), 105 stripes, 18 turquoise, 23 red, 22 blue, 23 yellow, 10 green, 9 white

(g) *Tabriz* (1984), 96 stripes, 14 turquoise, 21 red, 16 blue, 23 yellow, 11 green, 11 lilac

(h) *Not Riley* (2011) pattern generated following some but not all of Riley's implicit structure, 94 stripes, 20 turquoise, 20 red, 17 blue, 17 yellow, 10 black, 10 white

(i) *Red Devil* (1959) [Gene Davis], 65 stripes, 30 light red, 2 violet, 32 dark red, 1 lime green

(j) *Blue Gene* (2011) an application of Riley's Egyptian palette to Davis' structure, 65 stripes, 30 turquoise, 2 red, 32 blue, 1 yellow

Figure 2. Colour sequences used by (a)–(g) Bridget Riley, (i) Gene Davis, (h) & (j) the author. The original pieces represented by (a)–(g) and (i) are almost-squares about two metres on a side, with stripes of width about 19 mm (Riley) or 25 mm (Davis). Note that these are *not* the works of art nor are they copies or reproductions of the works of art. These serve only as a representation of the colour sequence within each work. Colours are close to, but not exact matches of, those in the artwork.

two blues, a purple, a red, and a yellow). After the fourth stage, Riley moved from the vertical stripe to a different geometric construction, introducing a strong diagonal element alongside the vertical one (the "zig" paintings). This was a deliberate reaction to having reached the end of what she felt she could do within the constraints of the stripe motif and it introduced another structural element to allow her to investigate colour in new ways.

The black bands in the first stage were used as "rhythmical accents to drive, check, contain and break the colour development." The white intervals were used as both "pauses" and "clearances"

(a) *Silvered* (1981)

(b) *Après Midi* (1981)

(c) *Summer's Field* (1982)

(d) *Serenissima* (1982)

(e) *Luxor* (1982)

(f) *Bali* (1983) — with deep green replaced by dark grey

(g) *Tabriz* (1984) — with deep green replaced by dark grey and lilac by light grey.

Figure 3. The black and white rhythms in Riley's compositions. This shows the colour sequences from Figure 2, with the four principal colours replaced by the same mid-grey. Black and white are unchanged.

to set free the "high flaming accents" of the bordering colours [12]. Riley's black and white articulation can be seen in Figure 3, where all of the colour has been replaced by a mid-grey. There is a rhythm to the stripes, but it is not a regular rhythm. This idea can be seen in Riley's earliest experiments. *Black & White Rhythm* (1980) [16, Plate 85] is an early experiment in which the black and white stripes alternate, as they do in *Silvered* (1981) (Figure 2(a)). Later works do not have pure alteration of black and white.

Riley's use of musical terminology exemplifies that her work is in rhythmical and musical in inception, rather than mathematical, diagrammatic or methodological. She does not think in terms of numbers and equations but "sees" relationships, proportions, and correspondences [12].

In terms of analysis methods, the work presented here is close to the analysis of music [3, 7, 18, 28], in that the same sort of tools are applied. It is also similar to the analysis of Jackson Pollock's work and Piet Mondrian's later compositions [25–27], in that it is analysis of a specific style that is not necessarily amenable to generalisation. It bears some relation to authentication work, such as Taylor's controversial authentication of Pollock [9] and Lyu et al.'s authentication of Bruegel [15], although my key aim is to ascertain whether there is a mathematical characterisation, with authentication being a secondary consideration.

3. Analysis methods

Four tools were investigated to ascertain whether they could characterise Riley's colour sequences: entropy, separation distance, auto-correlation, and direct visual observation. In the

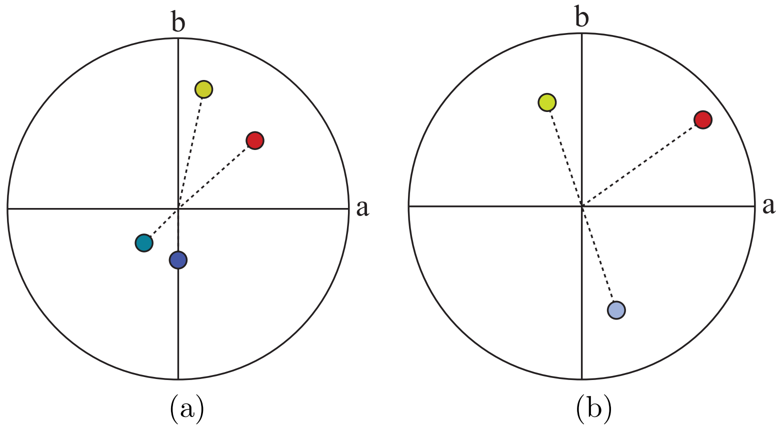


Figure 4. Colour palettes in *Lab* space. These diagrams show the colours projected onto the *ab* plane. *a* is the red-green axis, *b* is the yellow-blue axis. (a) Bridget Riley's Egyptian palette: two pairs of almost-complimentary colours (red-turquoise and yellow-blue), with one colour in one pair (turquoise) being close to one colour in the other pair (blue). (b) Gene Davis' *Red Devil* palette: a pair of complimentary colours contrasting with a colour orthogonal to the pair. N.B., these colours were not measured directly from the artwork, as the original art was unavailable. Each was measured from several different photographs, with the measurements checked against one another and the best measurements averaged. This means that the colour locations are only approximate.

first three cases, the source data is an ordered sequence of colours. Each distinct colour is allocated a unique index. This encoding of the artwork is possible because all of Riley's works have a fixed, limited palette of colours.

3.1. *Colour choice*

Before considering the sequence of colours, we must acknowledge that the selection of the colour palette is vital to the visual effect. One cannot just pick any set of colours. Riley's Egyptian palette comprises four colours of almost identical luminance: bright blue, turquoise, golden yellow, and ochre red. The fact that they are iso-luminant is a deliberate choice by the artist [12] and it causes interesting effects in the visual processing pathways in the human brain. The colour opponent theory of visual colour processing postulates three colour channels between the eye and the visual cortex: black-white (luminance), green-red, and yellow-blue [8]. Iso-luminant colours cannot be distinguished by the high-resolution luminance channel and so stimulate only the lower-resolution colour channels. This was exploited by Monet in *Soleil Levant* (1872) and is exploited by Riley here. Riley comments "...an important aspect of colour behaviour [is that] colour begins to interact if adjusted to a common tonal level" [12].

As well as being iso-luminants, Riley has chosen pairs of near opposites in colour space: red versus turquoise and yellow versus blue (see Figure 4(a)). These maximally stimulate the colour channels, causing visual shimmering along the edges where the opposing colours meet.

Finally, it is well known that colour perception depends on context (the phenomenon known as *simultaneous contrast* [23]). The particular relevance to the Egyptian palette is that the turquoise and the blue are sufficiently close in colour space that the human visual system can perceive a turquoise stripe in one part of the painting to be identical to a blue stripe elsewhere, if their contexts are constructed in the right way.

This palette was carefully chosen by Riley to create the effects that she wanted. In Figure 2(j), I apply Riley's palette to Davis' *Red Devil* (Figure 2(i)). This demonstrates that what is optimal in one configuration of stripes is not optimal in another. Davis' composition requires lighter and darker shades of the same colour (i.e., *only* luminance changes) and then two lighter opponent colours (Figure 4(b)). Naïvely combining Riley's palette with Davis' composition produces something that is less than the sum of its parts. This demonstrates that even the simple artistic space of stripe paintings is complicated in the location of its optima.

In passing, one may wonder why Riley chose four colours (plus black and white) rather than, say, three or five. Clearly, two colours would be too few. This would allow simply an alternating pattern of two colours, interspersed with black and white punctuation. If we wish to exploit complementary colours and avoid adjacent stripes of the same colour then a little experimenting will show that three colours does not provide a rich enough possibility for creating rhythms. Riley *did* use only three colours (red, green, blue) in her work of the 1970s, where the patterns were geometrically more complex and deliberately repetitive. Four colours thus seems to be the minimum to get an interesting effect when dealing purely with vertical stripes.

3.2. *Global entropy*

Entropy measures are generated by categorising features of the painting and then using the probabilities of their occurrence to calculate entropy:

$$H = -\sum_i p_i \log_2 p_i$$

I use three different categorisations:

Zeroth order entropy: each colour is treated as a separate category.

First order entropy: each possible pair of adjacent colours is treated as a separate category. The features of the painting are all the adjacent pairs of stripes, so there is one fewer pair than there are stripes.

Second order entropy: each possible triple of adjacent colours is treated as a separate category. The features of the painting are all adjacent triples of stripes, so there are two fewer triples than there are stripes.

3.2.1. Zeroth order

Zeroth-order entropy is simply a measure of the relative proportions of each colour. From observation, and from Riley's documentation, we know that the early stripe paintings (those that use the Egyptian palette) had roughly equal proportions of the four colours and that black and white occur about a third as often. That is, the proportions of the colours are 3:3:3:3:1:1, with black and white being the least likely colours. If these proportions were exactly right, then the zeroth-order entropy would be 2.45. Each artwork can be considered to be a sample of 100 stripes drawn from this probability distribution, so we should expect the zeroth-order entropy to be near, but not exactly, this number. As it happens, six of the seven early paintings that I considered are within 0.02 of this value (see Table 1).

3.2.2. First order

First-order entropy is a single number characterising what colours Riley allows to lie next to one another. Simple observation shows that a colour never lies next to a stripe of the same colour: that would create, visually, a double-width stripe, which cannot be allowed[1]. This means that Riley's compositions should have lower first-order entropy than patterns created randomly from the same distribution of colours.

When we plot the histogram of occurrences of each category used in first-order entropy, we find that certain other pairs of colours are also forbidden by the artist, and that these restrictions differ between paintings. Most obviously, black and white may never be adjacent. This is understandable given the rhythmic role of black-and-white as punctuation in the compositions. But we also find the absence of other colour transitions, shown in Table 1.

The table shows a progression in Riley's application of restrictions. The earliest stripe paintings do not allow red to lie next to yellow nor blue to lie next to turquoise. That is, Riley does not

[1] Although none of Riley's finished paintings have double-width stripes, at least one of her rough studies does [16, Plate 88]: *Rough Stripe Study with Blue, Turquoise, Yellow and Green* [1983]

Table 1. Statistics of a selection of Riley's stripe paintings. The first seven all use the Egyptian palette. The next two are later works, which replace black by green and, in the case of *Tabriz*, white with lilac. The final two are not by Riley and are included to show differences in entropy measures. H_0, H_1, and H_2 are the zeroth-, first- and second-order entropy measures respectively. *Restrictions* are those colour transitions that do not occur. That is, we never observe stripes of the two named colours lying next to one another, either left to right or right to left. The letters at left refer to those in Figures 2 and 3).

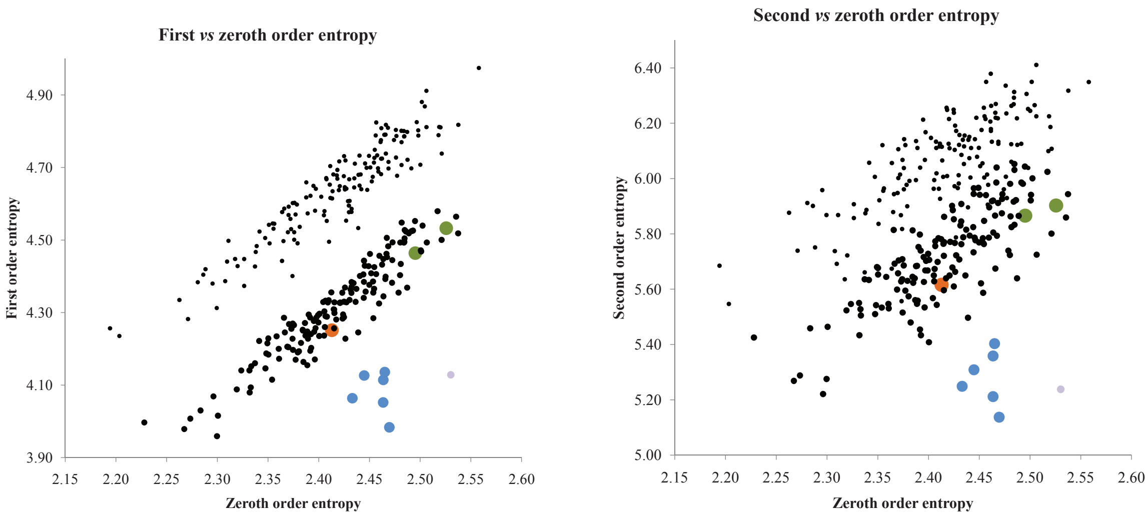
	Title	H_0	H_1	H_2	Restrictions
(a)	*Silvered* (1981)	2.46	4.11	5.21	black-white, red-yellow, blue-turquoise
	Winter Palace (1981)	2.46	4.05	5.36	black-white, red-yellow, blue-turquoise
	Silvered 2 (1981)	2.47	3.98	5.14	black-white, red-yellow, blue-turquoise
(b)	*Après Midi* (1981)	2.44	4.13	5.31	black-white, red-yellow, blue-red, black-red
(c)	*Summer's Field* (1982)	2.43	4.06	5.25	black-white, red-yellow, blue-red, black-red
(d)	*Serenissima* (1982)	2.47	4.14	5.40	black-white, red-yellow, blue-turquoise
(e)	*Luxor* (1982)	2.41	4.25	5.62	black-white, red-yellow
(f)	*Bali* (1983)	2.50	4.46	5.87	*none*
(g)	*Tabriz* (1984)	2.53	4.53	5.90	*none*
(h)	*Not Riley* (2011)	2.53	4.13	5.24	black-white, red-yellow, blue-turquoise, *not by Bridget Riley*
(i),(j)	*Red Devil* (1959)	1.27	1.60	1.70	*uses only four colours: two shades of red, violet and lime; by Gene Davis*

allow colours in the same quadrant of *ab* colour space (Figure 4) to lie next to one another. A different set of restrictions is evident in *Après Midi* (1981) and *Summer's Field* (1982). Here blue is allowed to lie next to turquoise, but more severe restrictions are placed on red, which is not allowed to lie alongside yellow, blue or black. *Luxor* (1982), by contrast, is more relaxed; here the only restrictions are that red must not lie next to yellow and the usual black-white prohibition. The later paintings (which use a different colour palette) have no first-order restrictions at all. We might expect all these differences to be reflected in the entropy values.

Figure 5(a) graphs zeroth- against first-order entropy. It shows 300 randomly generated patterns, the nine paintings by Riley from Table 1, and the example from Figure 1. The randomly generated patterns are of two types. Half of them are random patterns generated with the 3:3:3:3:1:1 distribution of colours used by Riley. These show a strong linear correlation between zeroth- and first-order entropy, as one would expect because, in the limit as the number of stripes goes to infinity, $H_1 = 2H_0$ if there are no constraints on which colours lie next to which other.

The other half of the random patterns are generated from the same distribution but also with the restriction that no adjacent stripes may be the same colour. This has a different linear relationship, with a lower first-order entropy for any given zeroth-order entropy than the unrestricted random distributions.

Six of the nine Riley paintings that I considered lie in a cluster to the lower right of the second random collection (blue dots). There are several interesting things to note. First, this cluster clearly reflects Riley's strict rules about forbidden adjacencies: the first-order entropy is lower for a given zeroth-order entropy than it would be if there were no additional constraints on adjacencies (i.e., Riley's paintings all lie below the randomly generated versions). Second, it shows that Riley is consistent in her proportions of colour (characterised by zeroth-order entropy) and in how she controls adjacency (characterised by first-order entropy). Indeed, the four paintings that have a zeroth-order entropy of 2.46 or 2.47 all use exactly the same restrictions on adjacencies, indicating a considerable consistency in proportions of colour, though with differing approaches to proportions of different type of adjacency, indicated by the range of values for first-order entropy. Third, it offers the tantalising thought that we could distinguish real from fake paintings using just this two-dimensional space, as has been attempted with Jackson Pollock's drip paintings [1, 14]. For example, the painting in Figure 1 is not by Bridget Riley and is constructed with slightly different rhythmic structure. It falls some way to the right (the lilac dot) of the Riley cluster, because, although the adjacency rules have been followed, it does not have the correct distribution of colours. By contrast, two of those six paintings (*Winter Palace* and *Silvered 2*) were selected for analysis after generating this hypothesis about where Riley's



(a) (b)

Figure 5. Graphs of (a) first- and (b) second-order entropy against zeroth-order entropy. The smallest black dots represent 150 randomly-generated patterns with a distribution of six colours in the rough proportions 3:3:3:3:1:1. The medium-sized black dots represent 150 randomly generated patterns with the same distribution of colours but with the added restriction that no colour may be adjacent to a stripe of the same colour. The four groups of large coloured dots represent different examples of paintings. The cluster of six blue dots (lower right) are all paintings from 1981 and 1982 that use the Egyptian palette. The single orange dot at $(2.41, 4.25)$ is *Luxor* (1982). The two green dots near $(2.5, 4.5)$ are works from 1983 and 1984. The small lilac dot at lower far-right is the pattern in Figure 1.

paintings lie in this space. It was pleasing to see that they lie very close to the other four blue dots in the cluster.

However, not all is rosy in the world of spotting fakes. This cluster is only characteristic of the early stripe paintings from 1981 and 1982 that use the Egyptian palette. Even then, at least one painting from that period fails the “Riley cluster test.” *Luxor* (1982) lies within the “random band”. It is near to the Riley cluster but quite distinct from that cluster. This is a problem for the mathematical characterisation. It means that this two-dimensional space does not allow us to distinguish a genuine Riley painting from randomly-generated fake. Worse, it implies that, in this two-dimensional space, *Luxor* is indistinguishable from some randomly-generated fakes.

To ascertain whether this mathematical characterisation might match human perception, I conducted an informal test into a human expert’s ability to distinguish real from randomly-generated paintings. I procured reproductions of four of Riley’s stripe paintings and produced seven randomly-generated paintings, printed so that it was not possible to distinguish between the real reproductions and the fakes except by consideration of the sequence of colours. Having spent considerable time investigating Riley’s stripe paintings, I expected to be able to easily distinguish the real from the fake. I identified all of the randomly-generated paintings as fakes and all but one of Riley’s paintings as genuine, but I dismissed *Luxor* as also being randomly-generated. This may be owing to my having an inadequate mental model of the features of a Riley stripe painting, but I find it odd that, having spent some time studying all the stripe paintings, including *Luxor*, I dismissed it as a fake. If we consider Figure 5(a), we see that *Luxor* could, indeed, be generated by chance by this random number generator. If we consider Table 1, we see that *Luxor* only has two restrictions, where all of the other early Riley paintings have three or four. This means that the entropy measure may be telling us something about Riley’s progression in compositional-style, rather than being some measure of authenticity.

As Riley developed her stripe paintings, she abandoned black in favour of a deep green (e.g., *Bali* (1983)) and then white in favour of a pale lilac (e.g., *Tabriz* (1984)). She also made two other changes. She allowed herself to put the deepest colour (green) next to the lightest colour (white or lilac) and she allowed any colour to be adjacent to any other. Our entropy analysis demonstrates two features of these two example paintings: there is a more even distribution of the six colours (the ratio in *Bali* is roughly 2:2:2:2:1:1 and that in *Tabriz* is 4:4:3:3:2:2), as indicated by the higher zeroth-order entropy, and adjacencies are much less restricted, as indicated by the higher first-order entropy.

At this point, you should be suspicious of the graph in Figure 5(a). The randomly generated examples use a 3:3:3:3:1:1 ratio of the colours. It could be that some of the Riley paintings appear to be in the randomly-generated region because they have a different ratio of colours. It is possible to test this. We can create an *expected* value of first-order entropy based on the actual distribution of colours in a given painting, rather than on some idealised distribution. Let the proportion of stripes of a given colour in a painting be represented by p_i, where i is the index for the colour, and let the proportion of a given pair of adjacent colours be represented by $p_{i,j}$, with i being the index of the colour of the leftmost stripe. The zeroth- and first-order entropies for a painting are calculated by[1]:

$$H_0 = -\sum_i p_i \log_2 p_i$$
$$H_1 = -\sum_i \sum_j p_{i,j} \log_2 p_{i,j}$$

We can calculate the expected first-order entropy, H_{1a}, when adjacent stripes are permitted to be the same colour as:

$$\begin{aligned} H_{1a} &= -\sum_i \sum_j p_i p_j \log_2 (p_i p_j) \\ &= 2H_0 \end{aligned}$$

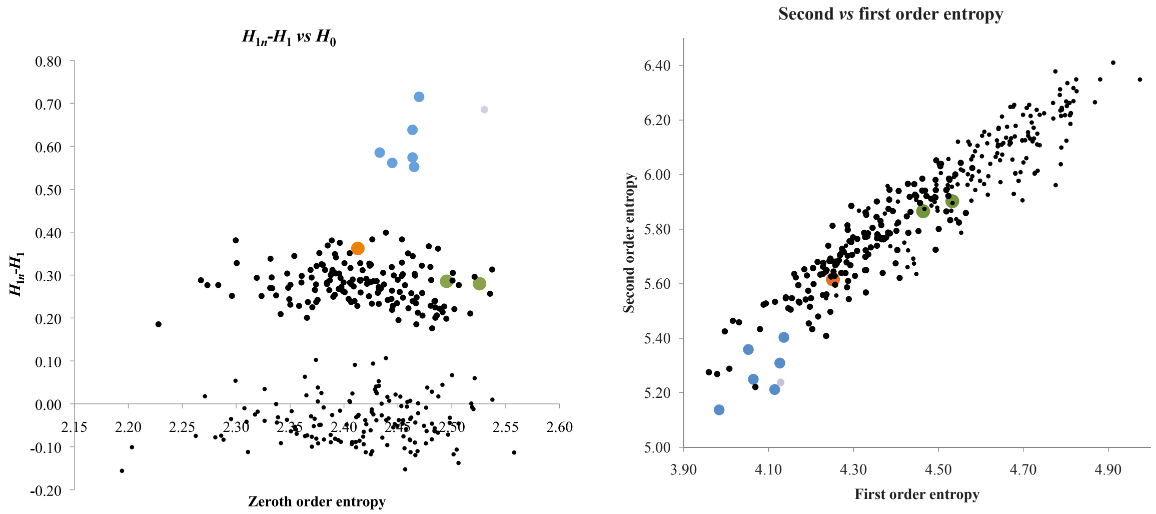
If we do not allow adjacencies, then we have alternative expected first-order entropy, H_{1n}:

$$p_{\text{tot}} = \sum_i \sum_{j \neq i} p_i p_j$$
$$H_{1n} = -\sum_i \sum_{j \neq i} \frac{p_i p_j}{p_{\text{tot}}} \log_2 \left(\frac{p_i p_j}{p_{\text{tot}}} \right)$$

Because $H_{1a} = 2H_0$, a plot of $H_{1a} - H_1$ against H_0 would simply give a skewed version of Figure 5(a). More interesting is the plot of $H_{1n} - H_1$ against H_0, which can be seen in Figure 6(a). The surprising result is that this difference of entropies does not reveal any extra information. The three Riley paintings (*Luxor*, *Bali*, *Tabriz*) that lie within the “randomly-generated band” (Figure 5(a)), remain in the randomly-generated band (Figure 6(a)), even when the true distribution of colours is taken into account. This means that the considered entropy measures do not permit us to distinguish some of Riley’s original paintings from randomly-generated versions.

Working from the other direction, it is also possible to generate colour sequences that exactly match one of Riley’s sequences, but which have dramatically different visual properties. Figure 7 shows two sequences with exactly the same entropy measures. The top sequence is that of Riley’s *Serenissima*. The bottom sequence is carefully constructed to exhibit features that do not appear in any of Riley’s paintings (see Section 3.3). Although both are constructed by a human agent, they are dramatically different in their visual impact.

[1]There are two conventions for numbering the orders of entropies. Some will consider what I call zeroth- and first-order entropy to be first- and second-order.



(a) (b)

Figure 6. (a) Graph of the difference $(H_{1n} - H_1)$ between first-order entropy and predicted first-order entropy assuming no adjacent stripes of the same colour. This is plotted against zeroth-order entropy for easy comparison with the graphs in Figure 5. (b) Graph of second- against first-order entropy, showing the strong correlation between the two.

(a) *Serenissima* (1981)

(b) a sequence created by hand to match exactly the entropy characteristics of *Serenissima*

Figure 7. Colour sequences of (a) *Serenissima* (1981) and (b) a sequence created to match the entropy characteristics of *Serenissima* but to contain visual features that are not found in Riley's paintings. Both sequences have $H_0 = 2.47$, $H_1 = 4.14$, and $H_2 = 5.40$.

3.2.3. Second order

Observation of the visual structure of Riley's paintings (Section 3.3) tells us that her structure depends on more than what colours she allows to lie adjacent to one another. We might therefore expect second-, third-, or higher orders of entropy to give more information. Let the proportion of a given triple of adjacent colours be represented by $p_{i,j,k}$, with i, j and k being the indices of the colours of, respectively, the leftmost, centre, and rightmost stripe in the triple. The second-order entropy is calculated by:

$$H_2 = -\sum_i \sum_j \sum_k p_{i,j,k} \log_2 p_{i,j,k}$$

Figure 5(b) shows second-order entropy graphed against zeroth-order entropy. This demonstrates that, while second-order entropy is not as strongly correlated to zeroth order for the randomly-generated patterns, it gives no further distinguishing features than does first-order entropy. Indeed, first-and second-order entropy are strongly correlated with one another for the 310 patterns graphed (Pearson's correlation coefficient, $r = 0.93$), as can be seen in Figure 6(b).

One possible reason is that these paintings contain only about 100 samples chosen from six different categories (the colours). This gives 36 different categories for first-order entropy and 216 different categories for second-order entropy. As there are more categories than samples for second-order entropy, there is likely to be insufficient data for second-and higher-order entropy to provide any additional information. Boon and Decroly found a similar problem with higher-order entropies in music analysis [3].

In conclusion, the considered entropy measures do characterise something about the art. They reflect something of the artist's development of her style. But they are insufficient to reliably distinguish her art from randomly-generated patterns.

3.3. *Visual evaluation*

Before we consider other mathematical tools, I will briefly discuss the visual features that appear to be consistent across Riley's compositions.

Riley's paintings have a rhythm to them, set by the white and black lines. Beyond that, Riley has been careful to ensure that there is no outstanding feature on which the eye can fix and concentrate on. This leaves the eye free to roam back and forth, picking up on the subtle rhythms.

By contrast, the example in Figure 7(b) has been deliberately designed to demonstrate features that Riley avoids. These are:

- Large expanses of just two colours alternating. Most of Riley's paintings have no more than four adjacent stripes of alternating colours. None have more than five. The addition of even a sixth stripe creates quite a striking pattern on which the eye tends to fixate.
- Black and white stripes adjacent. This would counter-act Riley's use of the black and white stripes as the structural underpinning of the composition.
- Black or white stripes too close to the nearest stripe of the same colour. In Figure 7(b), the triple white stripe stands out visually (there are three whites within just eight stripes). Riley does not completely eschew this, but when she uses it, she is careful. The obvious example among the paintings considered here is the three blacks within ten stripes in *Luxor*, which we have already seen is something of an outlier in her Egyptian-palette paintings.
- "Licorice allsorts", where the same colour lies either side of either a white or a black stripe. This can be remarkably obvious to the viewer. It occurs six times in Figure 7(b) and some of those occurrences are easy to spot. Riley does not avoid it completely and uses it in places where it does not leap out immediately at the viewer. It occurs twice in each of *Summer's Field*, *Serenissima*, and *Luxor* (Figure 2(c), (d), (e)) but one tends to have to hunt for those occurrences rather than easily spotting them.

3.3.1. Expert evaluation

As described in the informal test above (Section 3.2.2), I have spent considerable time looking at Riley's work, and I find that I am able, therefore, to distinguish between her compositions and random ones reasonably reliably, though not perfectly. I believe that I am doing this mostly by looking for the absence of the features listed above and looking for a rhythm to the painting.

3.3.2. Non-expert evaluation

It would be interesting to ascertain whether non-experts could distinguish between Riley's work and randomly-generated work. I conducted an informal pilot study on six participants from my own research group. They are all educated to degree level, aged 24 to 59, five male and one female. They were given ten stripe patterns of size 175 mm × 175 mm, printed on A4 paper, with printed colours matched as closely as possible to the Egyptian palette as reproduced in the Tate Museum's book of the 2003 Riley retrospective [16]. Six of the patterns were representations of Riley's paintings, four were randomly generated. One of the random patterns had some adjacent stripes of the same colour; the other three had no adjacent stripes with the same colour. Participants were shown the patterns one after another and asked to choose whether each

Table 2. Raw experimental results. A cross indicates that the participant correctly identified the pattern as either human-generated (top set) or randomly-generated (bottom set).

	Participant A	B	C	D	E	F	Correct (out of 6)
Riley's paintings							
Silvered		×			×	×	3
Summer's Field				×		×	2
Serenissima				×	×	×	3
Luxor	×	×				×	3
Bali		×		×	×	×	4
Tabriz			×		×	×	3
Randomly-generated							
with adjacent			×	×	×	×	4
without adjacent			×		×	×	3
without adjacent	×		×	×	×	×	5
without adjacent		×	×	×	×	×	5
Riley correct (out of 6)	1	3	1	3	4	6	
random correct (out of 4)	1	1	4	3	4	4	
total correct (out of 10)	2	4	5	6	8	10	

pattern was generated by a human artist or by a random number generator[1]. Each participant saw the patterns in a different order.

The raw results are given in Table 2. These serve to demonstrate only that there appears to be nothing obvious to non-experts that would distinguish a Riley painting from a randomly-generated pattern. From discussion with the participants, their decision method was either to guess or to generate their own hypotheses as to what constituted "random" and "human-generated". The participant with the lowest correct score ("A") had chosen to look for clusters of similar colours as evidence of the artist's hand. Such clusters are produced by the random number generator but are carefully avoided by Riley. Participant "A" thus identified all of Riley's paintings as "random", except for *Luxor*. Participant "E" chose to look at the pattern of black and white stripes, correctly classifying eight of the patterns from this. One participant ("F") correctly identified all ten patterns but, when pressed as to how, said simply that it was "educated guessing." By comparison, consider how accurate would be the entropy measures discussed in the previous section. For example, the simple (and incorrect!) criterion that a Riley painting could be distinguished by $H_{1n} - H_1 > 0.50$ would score 7/10: it would correctly identify all four random patterns as random but only three of the Riley paintings as human-generated.

In summary, Riley's paintings do have distinctive visual characteristics, but these are not obvious to a novice observer. However, the novice can be trained to identify them by looking for specific features, becoming an expert able to distinguish Riley from random with reasonable accuracy (cf. Hypothesis 1 in my paper on Riley's earlier work [5]).

3.4. *Separation distance*

Given that Riley has put structure into her compositions, it would seem sensible to consider some measure of the distribution of the colours. One way to do this is to look at the separation between adjacent stripes of the same colour. Example histograms of this statistic are in Figure 8.

Consideration of a large number of these histograms leads to the hypothesis that the most obvious features, which might distinguish Riley's compositions from randomly-generated ones, are the minimum distance between black stripes and the minimum distance between white ones. In Riley's Egyptian-palette compositions (the first seven paintings in Table 1) these minima are

[1]Note that I did *not* ask participants to judge which patterns were more attractive as it is unclear whether Riley was aiming for attractiveness in these compositions. See my comment in Section 2 on the difference between the beautiful and the sublime.

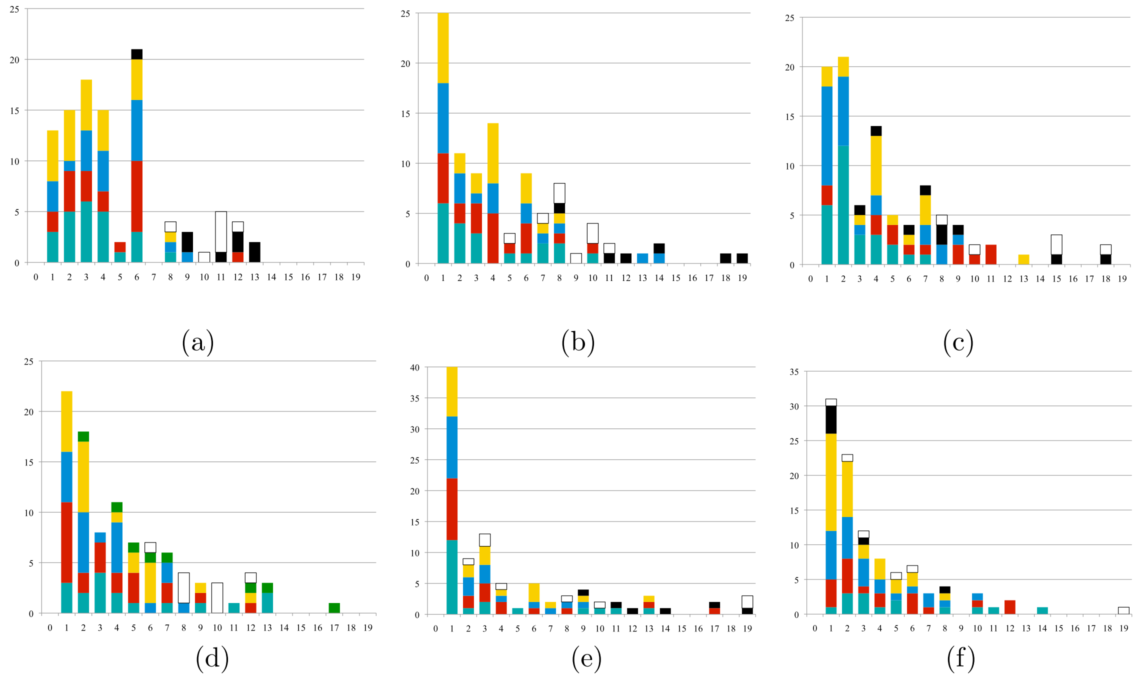


Figure 8. Histograms showing the number of stripes between adjacent occurrences of stripes of the same colour (the “separation distance” between occurrences). Note that no stripe is allowed to lie next to a stripe of the same colour, so the zero-distance column is empty. The four Riley paintings are shown to the same vertical scale. The two non-Riley patterns require a different scale. (a) *Silvered*, (b) *Serenissima*, (c) *Luxor*, (d) *Bali*, (e) the pattern from Figure 7(b) that matches the entropy characteristics of *Serenissima*, (f) a randomly-generated pattern with the 3:3:3:3:1:1 distribution of colours and no adjacent stripes of the same colour.

in the ranges 3–8 and 5–8 respectively.

If we consider our sample set of 150 randomly-generated patterns with no adjacencies, 84% have black stripes within four stripes of one another, with 76% for white stripes. Just under 10% have a minimum stripe separation of more than eight for one of black and white. However, if we construct a rule that will definitely catch all of Riley’s seven Egyptian-palette paintings (minimum black separation at least three and minimum white separation at least five), then only eight of the 150 randomly-generated patterns (5.3%) also pass the test.

It is therefore possible to construct a relatively simple mathematical test that allows distinction to be made between Riley’s paintings and randomly-generated patterns, with few false positives and no false negatives. The test aligns with one of the visual observations in Section 3.3 (that black or white stripes may not be too close to one another). However, it is somewhat artificial in its construction, being simply a test based on the data in seven particular paintings. Some more complex test could, obviously, be invented, probably by considering more of Riley’s catalogue of stripe paintings. However, such a test runs the risk common to machine-learning methods: that we will overfit our model to the small sample set that we have.

Of more interest is to consider other works. Riley’s later paintings, *Bali* and *Tabriz*, break the separation rule: *Bali* has dark green stripes separated by just two other stripes, while *Tabriz* has lilac stripes separated by just one other stripe. This is presumably because the move away from black and white to dark green and lilac allows those latter colours to act sometimes as punctuation and sometimes as colours in their own right.

The constructed pattern in Figure 7(b), although matching the entry statistics of *Serenissima*, has dramatically different distribution of separations (compare Figures 8(b) and (e)). In particular, notice the dramatic increase in identically-coloured stripes separated by just one other stripe. This reflects the fields of alternating colours in that pattern. Notice also that this pattern still maintains a significant separation between black stripes.

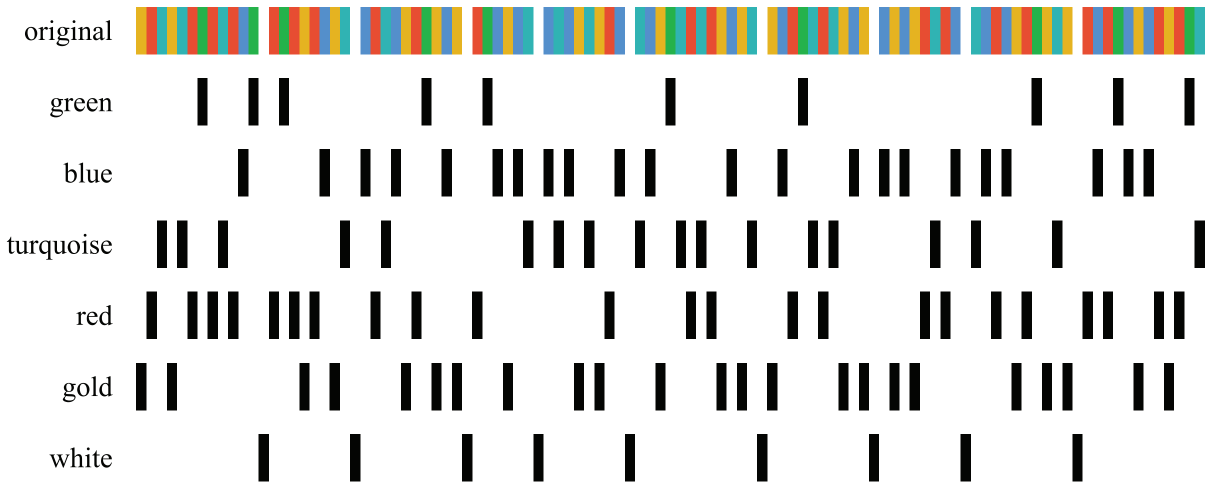


Figure 9. The locations of the individual colours in *Bali*, demonstrating that there is a rhythm to the composition, and that each colour is distributed fairly evenly across the composition.

Finally, consider one example of a randomly-generated pattern (Figure 8(f)). While this example is typical of the histograms of randomly-generated patterns, those histograms do vary considerably in character. The only notable common feature is that there are usually (with probability above 60%) white or black samples with separation of just 1 or 2.

3.5. *Auto-correlation*

Consider Riley's stripe paintings as visual music. The analysis techniques used for music may be appropriate in analysing the stripe paintings. The rhythms can be visualised by splitting a composition into its individual colours, as is done for Bali in Figure 9.

The two key differences between music and the stripes are timing and pitch. There is no clear connection between *timing* in music and horizontal position in a stripe painting. Figure 3 shows the black and white rhythms in some of Riley's stripe paintings. The rhythm is not exact in terms of horizontal position, whereas musical rhythm is precise in its timing. We should therefore expect that some musical analysis techniques would fail for the stripe paintings, where they assume that rhythms are regular. Musical *pitch* has its analogue in the stripes' colours. However, pitches are located in a univariate space, with a clear definition of absolute difference between two pitches, whereas colours are in a more complex perceptual space, and there is no one absolute difference measure than can be adopted.

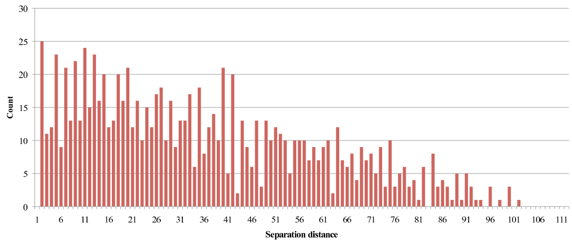
One similarity between music and Riley's composition is the concept of forbidden transitions. Hsu and Hsu [7] show that, in music, certain transitions are common and some are forbidden. For example, most folk music and much early classical music forbids the use of the transition between pitches of a diminished fifth. This has its analogy in Riley's forbidden colour adjacencies (Table 1).

Auto-correlation has been used in musical analysis [3]. We can apply auto-correlation also to stripe paintings.

For these patterns, we have nominal data rather than ordinal data, so our auto-correlation checks only that two stripes have the same colour, rather than considering any distance in colour space:

$$a(d) = \sum_i \delta(c_i, c_{i+d})$$

where $a(d)$ is the auto-correlation parameterised by a separation distance, d, c_i is the colour category of stripe i, $\delta(i, j)$ is the Kronecker delta: $\delta(i, i) = 1$, $\delta(i, j) = 0, i \neq j$.



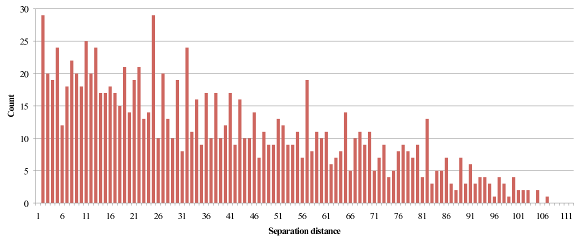


Figure 10. Graphs of auto-correlation for (a) *Serenissima* and (b) a randomly-generated pattern with six colours in the rough proportions 3:3:3:3:1:1 and no colour adjacent to a stripe of the same colour.

Two example histograms are shown in Figure 10. Because there are a limited number of stripes, the maximum possible value of $a(d)$ decreases linearly with d. Therefore, we would expect a completely random pattern to produce a histogram of $a(d)$ against d that is triangular. If there are clear repetitive patterns, we would expect to see strong peaks and troughs in the histogram. For example, *Serenissima*, *Summer's Field*, and *Silvered 2* all exhibit $a(d)$ values at $d = 2, 5, 7$ that are roughly double the values at $d = 3, 4, 6$. The later works, *Bali* and *Tabriz*, also exhibit peaks at $d = 2, 5$, though the difference is not so emphatic against the surrounding values of d. In practice, however, the noise in the auto-correlation plots swamps any peaks and troughs making it difficult to differentiate a Riley painting from a randomly-generated pattern. This is likely because Riley's stripe patterns lack a *regularly-spaced* rhythm. There is nothing here that gives a cut-and-dried mathematical method of distinguishing the artistic work from randomly-generated patterns.

3.6. *Local entropy*

We need a more flexible tool for analysing rhythmic structures. Voss and Clarke demonstrated that, averaged over long time scales and multiple pieces, music has a $1/f$ spectrum [28]. Going to a smaller time scale, Nettheim showed that, averaged over just a single movement, music has a spectrum closer to $1/f^2$ [18]. Boon and Decroly's more detailed analysis at this time scale shows that the behaviour in single movements of classical Western music is $1/f^\nu$, $1.79 < \nu < 1.97$ [3]. Of relevance to our problem is their comment about the even shorter, local, time-scale, which they say "...characterises the dynamics of a musical cell, i.e., a group of ten to twenty successive notes or sounds which are highly correlated in time. Such short time correlations are found in almost any music...and are therefore not very useful for a quantitative analysis designed to characterise works, composers or styles."

While such local analysis is not helpful in music, a local analysis might help in our stripe paintings. This is motivated by a consideration of Figure 9, which demonstrates a fairly even distribution of each colour, and no dramatic clumping. That is, in order to produce a visually appealing effect, I hypothesise that Riley is constrained to prevent any particular feature dominating at any point in the image: the eye must be free to range over the composition without getting stuck on any one thing.

To analyse this property, consider *local entropy*. That is, calculate the zeroth-order entropy in a local window, then examine how that statistic varies as the window moves across the composition. This measure turns out to be reasonably successful.

Given a window width, w, we calculate the zeroth-order entropy for w adjacent stripes. We repeat this for all possible sets of w adjacent stripes, giving $N - w + 1$ entropy measures for a composition with N stripes. Figure 11 plots summary statistics for these entropy values for thirty compositions. It shows upper and lower quartiles, maximum, and minimum for each composition, for $w = 24$. Informally, we can observe that the 21 randomly-generated compositions, in general, demonstrate a larger *overall range*, larger *inter-quartile range*, smaller *minimum*, and smaller *lower quartile* than do Riley's compositions.

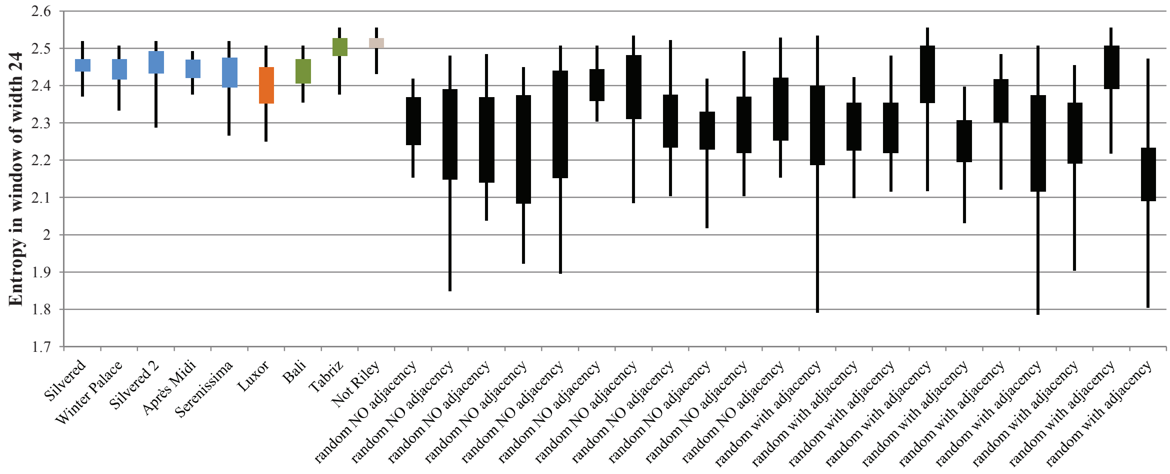
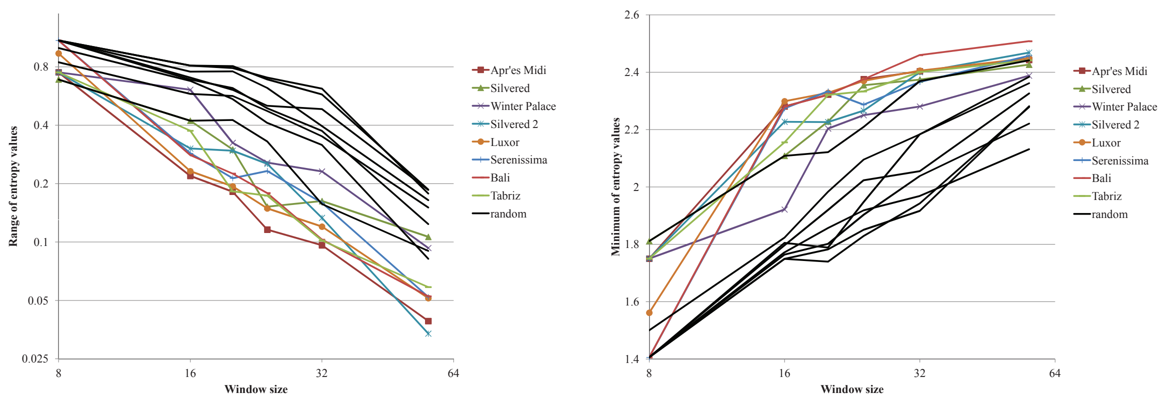


Figure 11. Local entropy statistics for eight Riley compositions, one composition by the author, and 21 randomly-generated compositions. The bars show minimum, lower quartile, upper quartile, and maximum values of locally windowed entropy across the composition. The window size is $w = 24$. The colours match those in Figures 5 and 6.



(a) (b)

Figure 12. (a) Graph of the *overall range* of local entropy values against window width for eight Riley compositions (coloured lines) and eight random compositions (black lines). The window widths used are $w = 8, 16, 20, 24, 32, 56$. (b) Graph of the *minimum* local entropy value against window width for the same sixteen compositions.

I tested these four statistics for eight Riley compositions and 150 randomly-generated compositions at $w = 8, 16, 20, 24, 32, 56$. Figure 12 plots how two of these four statistics (*overall range* and *minimum*) vary with window size for the eight Riley compositions and eight exemplars of the random compositions. Most of the random compositions are well separated from the Riley compositions but, on each measure, some random compositions do better than Riley's. It is clear that the measures change as the window size increases. The smallest window sizes are poor discriminators between Riley and random compositions: this is owing to the normal fluctuations in Riley's stripe choice dominating the bigger picture. The largest window sizes are also poor discriminators because the random fluctuations in the random compositions are smoothed out.

The best overall discriminator is *overall range* at $w = 24$. This is selected as the statistic and window size for which the Riley compositions are all closest to one end of the spectrum of values for that statistic. For *overall range* at $w = 24$, Riley's eight compositions were placed in positions $1, 2, 3, 4, 5, 7, 12, 13$ in an ordered list of 158 compositions. If we construct a rule that will catch all of these eight Riley compositions[1], then only five of the 150 randomly-generated patterns (3.3%) also pass the test.

[1]In this case, a position in the list of 13 or less is equivalent to an *overall range* below 0.257.

(a) a randomly-generated composition that matches well the local entropy statistics of Riley's compositions

(b) a randomly-generated composition whose local entropy statistics compare poorly with those of Riley's compositions

Figure 13. Colour sequences of randomly-generated compositions that do (a) well and (b) badly on the local entropy statistics of minimum, lower quartile, overall range, and inter-quartile range.

It is instructive to consider randomly-generated compositions that do well or badly on these statistical measures. Figure 13(b) shows a random composition that performs badly on all the statistical measures, compared with Riley's compositions. Notice the large expanse at left with neither black nor white stripes and the dominance of yellow stripes in various parts of the composition.

Figure 13(a), by comparison, shows a random composition that is the best of the 150 randomly-generated patterns. Indeed, it performs better than *Luxor* on all four statistics, and it performs better than *Serenissima* and *Silvered 2* on *overall range* and *minimum*. Notice that, despite scoring highly, it exhibits features that mark it out as not a Riley composition: the most immediately obvious are the three incidences of the "forbidden" black-white adjacency and the white-yellow-white triple toward the right hand side.

This indicates that there exist higher-level considerations that are not captured by local entropy. Again note that *Luxor* is an outlier in this space, as it scores worse on all four measures than two of the randomly generated compositions.

Particular note should be given to the two later compositions, *Bali* and *Tabriz*. Global entropy (Section 3.2) could not distinguish them from randomly-generated compositions. Local entropy clearly identifies them as having a regular structure, which we can interpret as a strong rhythm. This indicates that, when Riley freed herself from using black and white, she did not *abandon* the rhythmic structure of her compositions. She has maintained that rhythmic sense and, in the case of *Tabriz*, which has a very narrow range of local entropy values, she seems to have strengthened her rhythmic effect.

4. Conclusion

My aim was to ascertain whether there are mathematical measures that can characterise Bridget Riley's stripe paintings. This was motivated by three main considerations:

(1) Stripe paintings are an incredibly constrained art form, therefore it should be relatively straightforward to ascertain whether or not there is a mathematical characterisation.
(2) Bridget Riley's approach to composition is methodical and thoughtful. We can thus assume that her paintings are carefully constructed rather than random.
(3) Riley's paintings can appear random on a first glance, in contrast to works which have obvious structure, like Davis' *Red Devil* (Figure 2(i)). However, Riley's works have an underlying structure, in contrast to some other stripe works that are more random, like Davis' impulsive paintings [21], therefore Riley's works are challenging to characterise because they are close to random while not actually being so.

I have investigated four mathematical measures: global entropy, minimum distance, auto-correlation, and local entropy. Global entropy is able to distinguish some classes of Riley's stripe paintings, but not others.

Minimum distance provides more numbers to work with, but leads to the problem that it is all too easy to over-fit a numerical model to a limited number of paintings. We could probably successfully characterise Bridget Riley's paintings by combining enough of these numbers in some way. However, we would likely find that this was equivalent to simply checking whether a given pattern matched one of the known paintings.

Auto-correlation fails because the rhythmical patterns in Riley's paintings are irregular.

Local entropy performs well. With $w = 24$, we could provide thresholds that allow us to discard almost all of the randomly-generated patterns as fakes. Local entropy also demonstrated that there is more to Riley's compositions than can be characterised by a few numbers.

We can look at these results in two ways. On the one hand, we can say that there are indications that Riley's compositions can be characterised mathematically in ways that identify the underlying features. However, it is not a clear cut case. This leaves a quandary with how to extend this work to more complicated patterns: if we cannot adequately characterise stripe patterns, can we reasonably try to characterise patterns of more complexity?[1]

On the other hand, we can interpret these results as telling us something about Riley's art. For example, it appears that, over the course the early 1980s, Riley's patterns moved from more regular to more random (as characterised by global entropy), without losing their rhythmic structure (as characterised by local entropy). This reflects Kudielka's description of her artistic development [12]. This interpretation means that our mathematical characterisation is not a failure to identify Riley's paintings, but rather is a reflection the artist's intention. We thus have no new theories of human perception or of æsthetic measure, but instead some mathematical measures that tell us, in numbers, what the artist has already told us, in words.

Acknowledgements

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[1]One could argue that, if Riley's compositions could be characterised by only a few numbers, we would be disappointed in her as an artist.

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