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Regularity and randomness in Bridget Riley's early Op art

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Abstract

I investigate the trade-off between regularity and randomness in Bridget Riley's early Op art, focusing on White Discs 2 *(1964) and* Fragment 6/9 *(1965). I build on this to investigate the trade-off more generally. I analyse these two works and undertake three experimental investigations based on my observations. I first consider different types of randomness and the effect they have on the generated artwork. I then look at whether the introduction of randomness can be left to the computer or needs the artist's direction. For best æsthetic effect, there is some evidence that the choices made are not truly random. Finally, I consider how much randomness needs to be added to a regular pattern in order to produce a work that balances regularity and randomness in an æsthetically pleasing way. There is evidence that around two-thirds of the pattern needs to be retained.*

Categories and Subject Descriptors (according to ACM CCS): I.3.8 [Computer Graphics]: Applications I.3.m [Computer Graphics]: Miscellaneous – visual arts J.5 [Arts and Humanities]: Fine Arts

1. Introduction

Bridget Riley's *Fragment 6/9* (Figure 2) stimulated an investigation into the trade-off between regularity and randomness in art of this type. This paper was inspired by the question "what makes this composition æsthetically pleasing?" This led to three initial investigations into the æsthetic trade-off between regularity and randomness in these simple compositions. In particular, addressing the question of whether there is some balance between these two factors that produces the most æsthetically pleasing composition.

Op art consists of simple shapes in precise geometric relationships. Many examples are purely deterministic (Figure 1). By contrast, the works analysed and created in this paper add a minimal amount of randomness to a regular pattern. As such, I hypothesise that they are a useful testbed for experimenting with the æsthetic trade-off between regularity and randomness.

I report on my analysis of the artwork, my attempts to emulate the artistic result algorithmically, and my hypotheses on the nature of the trade-off between regularity and randomness that creates a pleasing æsthetic effect. These results are only preliminary; their function being to give insight into the types of question that should be asked in a more formal investigation.

2. Riley's early Op art

Bridget Riley (born 1931) studied art at Goldsmiths College (1949–52) and the Royal College of Art (1952–55) in London. She started investigating Op art in 1960. Her output from 1961 to 1966 consists of black-and-white geometric work and variations of this in shades of grey. From 1967 onwards she used colour, but always her work has restricted itself to a simple vocabulary of abstract shapes: squares, circles, ovals, lines, stripes, curves [Moo03]. I consider only work from the 1961–66 period, in particular the 1964 piece *White Discs 2* and the 1965 piece *Fragment 6/9*.

Riley's earliest Op art was purely deterministic, with no random elements. The geometry is described by the artist and the work simply an implementation of that geometry. It is therefore straightforward to write PostScript code [Ado90] to emulate these early works (Figure 1). Indeed, it is striking just how short the PostScript code needs to be in order to generate these works: an indication of how simple the geometric relationships are.

Some of Riley's later black-and-white works contain seemingly-random elements. In particular *Fragment 2/10*, *Fragment 6/9*, and *Fragment 7/5* from the *Fragments* series [Ril03], and *White Discs 2* (Figure 3). In the corresponding computer renditions, the positions of the geometric com-

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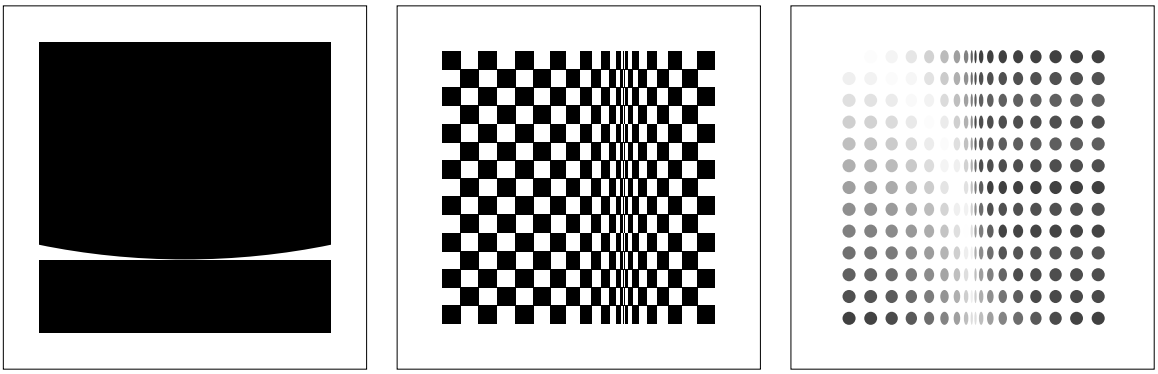

Figure 1: *Three computer renditions based on early Riley works. From left, the three renditions are based on* Kiss *(1961),* Movement in Squares *(1961) and* Loss *(1964). In all cases the computer rendition is similar to but not an accurate reproduction of the actual artwork. The PostScript code is of size 295, 467, and 519 bytes respectively,* ©*2008, Neil A. Dodgson, used with permission.*

ponents have to be explicitly stated in order to reproduce Riley's works. This raises questions of whether the same æsthetic effect could be generated by a pseudo-random algorithm, rather than explicit positioning.

3. Regularity and randomness

Consider both *Fragment 6/9* (Figure 2) and *White Discs 2* (Figure 3). At first sight, these appear little more than a random jumble of variously sized black discs on a white background. A little more study, however, will reveal hints of an underlying regular pattern. In both works, Riley strikes a balance between regularity and randomness.

Human beings are good at spotting patterns: regular, repeating features. However, we live in a world where things are not perfectly regular and so we are also good at coping with variation and randomness. An example from computer graphics is jittered and Poisson-disc sampling methods [Gla95], where the ability to handle randomness is used to trade-off structured aliasing (a regular pattern) for noise (a random effect). A useful example from nature is the structure of trees. All trees of a particular species are, from one point

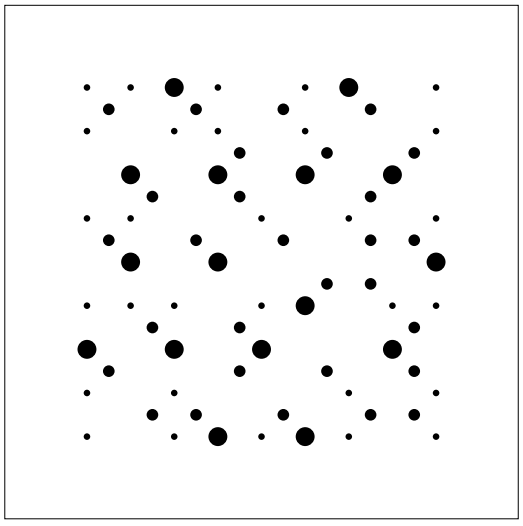

Figure 2: *A computer rendition based on* Fragment 6/9*, created using a PostScript program of 1670 bytes,* ©*2003, Neil A. Dodgson, used with permission.*

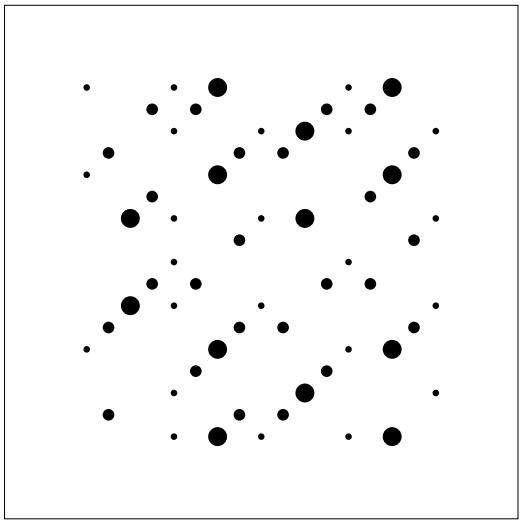

Figure 3: *A computer rendition based on* White Discs 2*, created using a PostScript program of 1670 bytes,* ©*2008, Neil A. Dodgson, used with permission.*

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	Small	Medium	Large	Total
deleted	13	7	5	25
remaining	23	25	13	61
total	36	32	18	86
proportion deleted	36%	22%	28%	29%

Table 1: *The number and proportion of discs deleted from the regular figure to create* White Discs 2.

	Small	Medium	Large	Total
unchanged	22	23	9	54
moved horizontally	7	2	6	15
moved vertically	–	3	1	4
deleted	1	4	–	5
inserted	1	1	–	2
proportion changed	29%	30%	44%	33%

Table 2: *The number of discs operated on in various ways to get from the regular figure to* Fragment 6/9.

of view, identical: their leaves have the same shape, their bark has the same structure, they all branch in the same way. But there is also great variation in trees: you do not find two specimens that are perfectly identical. This does not stop us from immediately identifying a tree as a tree and, with a little expert knowledge, distinguishing one species from another. The same is true of all natural things: wherever there is regularity, it is enlivened by randomness. Humans find perfect regularity to be unnatural. On the other hand, we find perfect randomness to be meaningless. In nature, we expect there to be a balance between regularity and randomness.

This brings us back to these two works. There is both regularity and randomness here. There is sufficient regularity to please our desire for order and sufficient randomness to make the composition interesting. The artist's skill seems to lie in selecting the right amount of randomness to apply, in order to make an effect which we find æsthetically pleasing.

To test the idea of a "right amount of randomness," I informally experimented with more ordered and more disordered variants of Riley's compositions. While it is a subjective judgement as to which is the more attractive, the analysis and experimentation throw some light onto which proportions of order make the more æsthetically pleasing compositions and which hypotheses need to be tested in future, formal experiments.

4. Analysis of *White Discs 2* and *Fragment 6/9*

To be able to undertake any experiments at all, I had to find an underlying order in Riley's compositions; you may like to try this yourself before reading any further. It is easy to see that the discs come in just three sizes (call them large, medium, and small) and that they are located at the corners of a regular diamond-shaped grid (this grid is most obvious in Figure 4). Furthermore, the medium discs appear exclusively on every even line, with the odd lines consisting exclusively of large and small discs. From this starting point, I go on to consider the two works individually.

4.1. *White Discs 2*

It seems clear that the pattern in *White Discs 2* (Figure 10 top left) is generated by removing discs from a particular regular pattern (Figure 10 bottom left) or, equally, by superimposing white discs on top of some of the black discs in the regular pattern.

Figure 10 breaks the work down into the three different disc sizes to demonstrate the regularities and irregularities at each level. The number of discs removed is tabulated in Table 1. This shows that about 30% of the discs are removed from the regular pattern to create the artwork. I consider the significance of this proportion in Section 5.3.

4.2. *Fragment 6/9*

Fragment 6/9 is more challenging. Indeed, it seems to be the end of a sequence of Riley's work that runs from the purely regular, through *White Discs 2* (1964), to *Fragment 6/9* (1965). This observation allows us to infer that the diagonal pattern of five discs, small–medium–large–medium–small, is also important in the later work. There is, however, insufficient evidence in the artwork to allow us to be certain

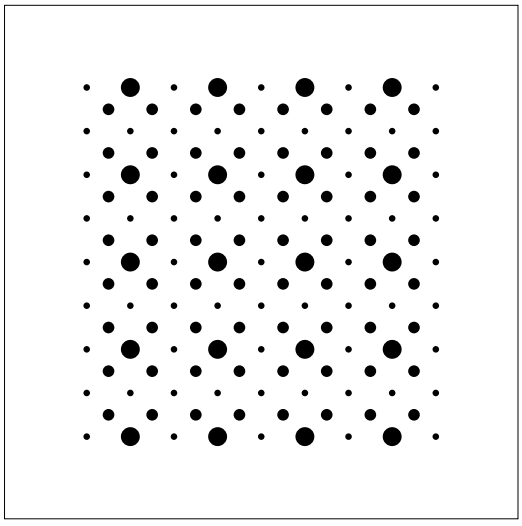

Figure 4: *A purely regular variant with considerably more discs than either* White Discs 2 *or* Fragment 6/9*,* ©*2003, Neil A. Dodgson, used with permission.*

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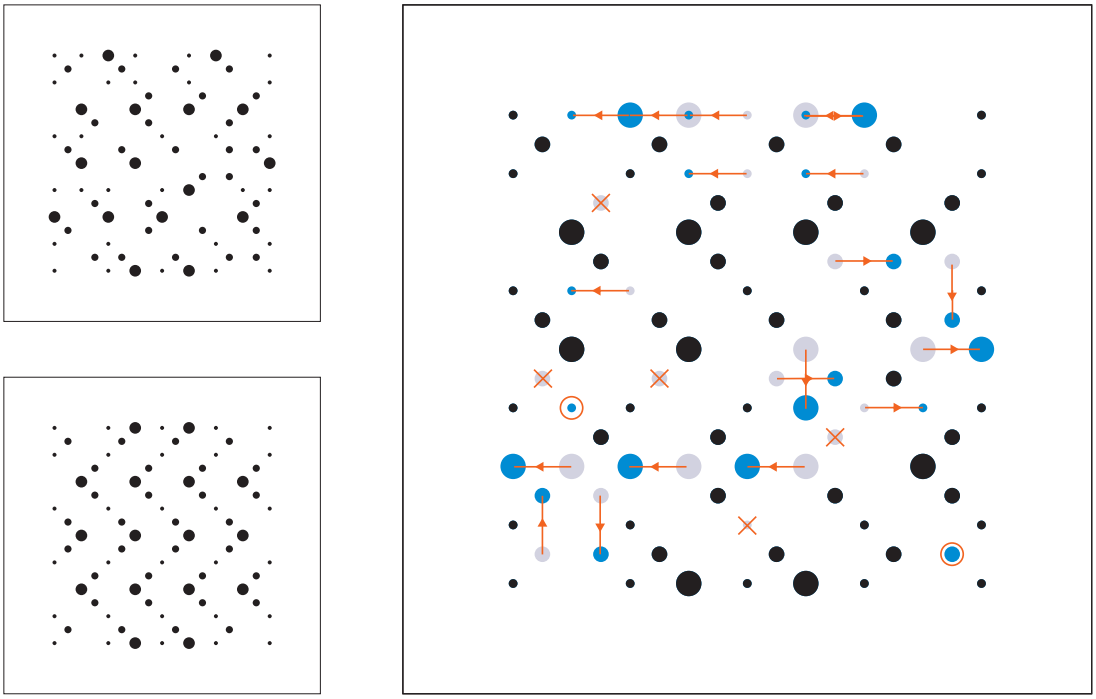

Figure 5: *Top left: a computer emulation of* Fragment 6/9 *with 75 discs. Bottom left: a regular variant with 78 discs. Right: how to get from the regular variant to the artwork with minimum effort; 54 discs remain unchanged, 19 discs move one unit horizontally or vertically, 5 discs are removed, 2 discs are added.* ©2008, Neil A. Dodgson, used with permission.

of recovering Riley's exact underlying regular structure, assuming that one ever existed.

Nevertheless, in order to undertake experiments, I needed to construct some plausible regular version. Working from my observations, I constructed two regular variants. In Figure 4, I have taken the observation that the large discs appear to be almost on a regular square grid, and have filled in the gaps with small and medium-sized discs on the appropriate lines. This produces a pattern with about twice as many discs (145 discs) as *Fragment 6/9* (75 discs) and, indeed, is also a super-set of the discs in *White Discs 2* (61 discs). I then remove discs from Figure 4 to get roughly the same number of discs as in *Fragment 6/9*, while maintaining regularity. Using the observation that there is some regularity in the diagonals in *Fragment 6/9*, I produced Figure 5 (bottom left). Figure 11 shows the construction of both the regular variant and the work itself from the three different disc sizes, demonstrating the regularities and irregularities at each level.

You can get from the regular version (Figure 5 bottom left) to the artwork (Figure 5 top left) with very simple operations (Figure 5 right): moving a disc one unit either horizontally (15 discs) or vertically (4 discs), removing a disc (5 discs) or adding a disc (2 discs). A breakdown of operations is given in Table 2. This shows that about two-thirds of the discs remain unchanged from the regular pattern, with one-third being moved, removed or added. This one-third figure is close to the 30% of deletions for *White Discs 2*. I consider whether this is significant in Section 5.3.

5. Experimental Investigations

The informal experiments took several forms. I report on experiments with producing variants of *Fragment 6/9* with varying types of randomness, with the sensitivity of *White Discs 2* to the particular discs that are removed, and to the æsthetic effects caused by removing various proportions of discs from a regular pattern.

5.1. Various types of randomness and regularity

I generated four types of variant of *Fragment 6/9* by applying different levels of constraint to the discs' locations (Figure 6). Each generated sample image consists of roughly the same number of discs of each size as are in the original artwork (Figure 2). There are two samples of each type of variant. From left to right in Figure 6, in increasing order of regularity and decreasing order of randomness, the four variants are:

Purely random. The discs are placed at random locations within a square; they may overlap. Any pattern that you perceive is entirely of your own imagining.

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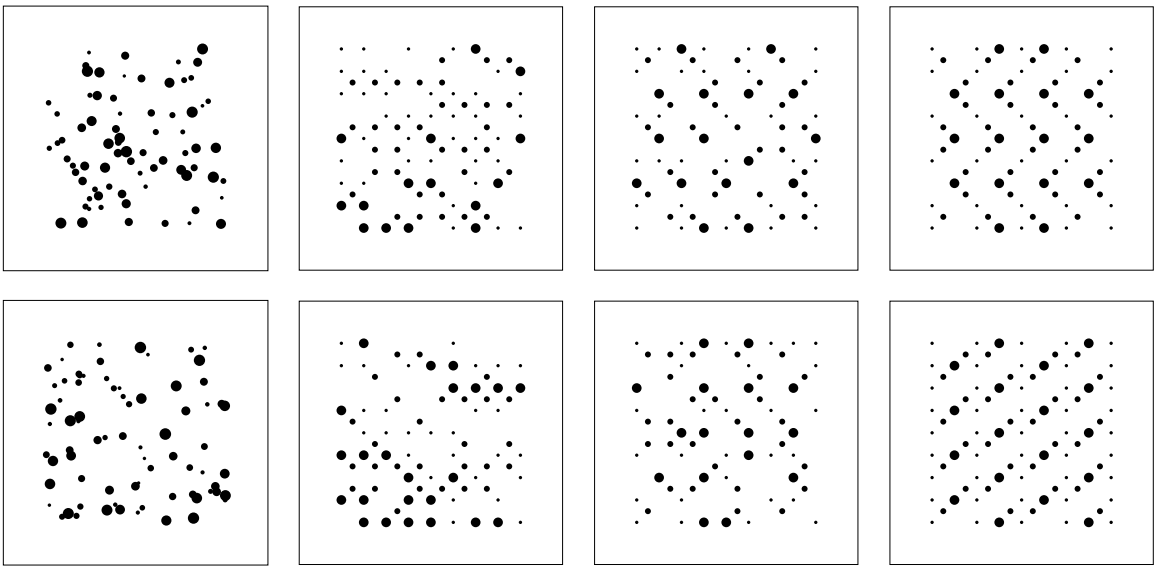

Figure 6: *Eight variants with varying degrees of randomness. The two sample patterns in each column were generated in similar ways. From left: purely random, constrained random, random moves from a regular base, regular. All variants are generated with PostScript programs of length between 800 and 1800 bytes, ©2003, 2008, Neil A. Dodgson, used with permission.*

Constrained random. The discs are placed on the grid with the constraint identified at the end of Section 3, so that small and large discs may appear only on odd lines, while medium discs appear only on even lines.

Random moves from regular base. The discs start in the regular arrangement shown in Figure 5 (bottom left). About a third of the discs are manipulated by moving them one unit, deleting them or inserting new discs, in roughly the proportions used by Riley (Table 2). The top example is a computer rendition of *Fragment 6/9*, created using the process shown in Figure 5. The bottom example is generated using a pseudo-random number generator. The algorithm for doing this is, by far, the most complex algorithm used in the paper. The output of the algorithm was fed into the PostScript program that actually does the drawing, so the size of the 1800 byte PostScript program does not reflect the complexity of the generating algorithm.

Regular. The discs are placed in a regular pattern.

Returning to the tree analogy (Section 3), the question is whether the two samples in each column are clearly of the same "species" as one another, while clearly of different species to the samples in other columns. While any æsthetic comparison of these images is, of necessity, subjective, I make two observations. First, the completely random variants appear to me to be of a dramatically different species of pattern to the three constrained variants, which in turn appear to be distinct "sub-species." Secondly, the fact that the Riley sub-species (Figure 6 third column) can be distinguished from the less constrained sub-species (Figure 6 second column) is evidence that there really is an underlying pattern to *Fragment 6/9*.

5.2. The randomness of the deleted discs' positions

The artistic intent comes entirely from the human artist; the computer is merely an aid to implementing that intent. In this experiment, I consider whether it is sufficient for the artistic intent to be to remove a certain proportion of discs from a regular pattern, or whether the intent needs to be more specific about which particular discs are removed.

The experiment was carried out on the regular pattern that underlies *White Discs 2* (Figure 6 bottom right). I randomly remove 30% of the discs from the regular pattern. The results of four such removals are shown in Figure 7. Again, æsthetic comparison of these images is subjective. I invite you to consider which, if any, of the four images is the most visually attractive. A superficial inspection may leave the impression that there is little to choose between the four. My judgement is that the central two are less balanced, visually, than is Riley's work, which is the leftmost of the four images. This implies that the artistic intent *is* more specific about which particular discs are removed than just a straightforward random removal of a certain proportion of the discs. If there is still a desire to produce an algorithm that emulates the artistic intent automatically, then the least that needs to be added is some measure of "balance." This needs further, more formal, investigation.

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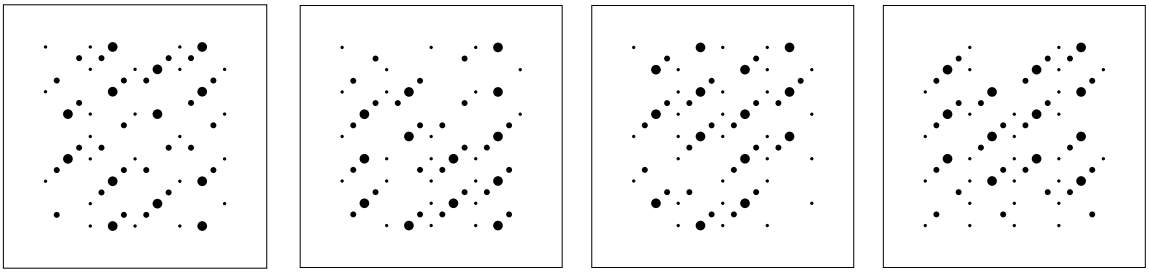

Figure 7: *Four examples showing removal of around 30% of the discs from the regular pattern in Figure 10 (bottom left). The leftmost example is a computer representation of Riley's* White Discs 2. *The other three are generated by a PostScript program of 1869 bytes, using PostScript's inbuilt pseudo-random number generator with three different seeds,* ©*2008, Neil A. Dodgson, used with permission.*

5.3. Varying the proportion of deleted discs

My final experiment investigated whether there is anything significant about the 30% deletion rate used by Riley (Section 4.1). Figure 8 demonstrates the removal of a proportion of the discs ranging from removal of 70% of the discs to removal of no discs. I applied the same process to a work inspired by Damien Hirst's lithographic print *Valium* [Hir00], which was itself inspired by the mathematical properties of the Fibonacci spiral that occurs naturally in the sunflower [Cox72, Vog79]. Figure 9 demonstrates the removal of a proportion of the discs from that pattern ranging from removal of 70% of the discs to removal of no discs.

Again, any analysis of these results is subjective. In both cases, I observe that removal of 50% or more of the discs destroys the regular pattern. There is insufficient of the pattern left for the brain to easily spot that there is an strong underlying regularity to the disc pattern. At the other end of the spectrum, removal of less than about 25% of the pattern leaves a "pattern with holes". The pattern detection in the human brain is able to complete the pattern easily and we see simply an incomplete version of the whole pattern. The removal of between about 25% and about 50% of the discs produces a result which has sufficient regularity for the underlying pattern to be discernible and sufficient randomness for the work to be viewed as a work in its own right, rather than an imperfect version of the pattern. This simple evidence indicates that there *is* something important about Riley's artistic decision to adjust around one-third of the discs in both *White Discs 2* and *Fragment 6/9*. Again, this hypothesis needs further, more formal, investigation.

6. Conclusions

Op art tends to be extremely regular. The particular examples examined here have introduced very limited randomness and yet even this limited randomness provides a great deal of complexity. This is a tribute to the pattern-detection systems in the human brain.

I conducted three preliminary experiments to help frame the direction of future research. The first experiment suggests the hypothesis that different algorithms, which balance regularity and randomness in different ways, create results that are distinguishable, by a human, as being of different "species," with examples generated by the same algorithm recognisable as being of the same species. The second experiment raises the hypothesis that even the simplest introduction of randomness can benefit from human judgement in order to get the "balance" of randomness right. The third experiment produces the interesting hypotheses that humans can easily detect patterns when up to about 25% of the pattern is removed or disturbed, that removal of over about 50% of the pattern destroys it, that there is an æsthetically interesting region between these two values, and that a good, artistic, balance between regularity and randomness is achieved by retaining about two-thirds of the pattern, while manipulating the other one-third in some way. In all three cases, the results are preliminary, based on informal observation only. The next step is to construct formal experiments to assess whether these hypotheses can be verified.

Acknowledgements

Thanks to Prof. Peter Robinson for creating the images in Figure 9 and to Emmanuel College, Cambridge, for hanging their copy of *Fragment 6/9* in their main committee meeting room, in which the author has spent so many hours.

About the artwork

White Discs 2, Bridget Riley, 1964, emulsion on hardboard, 104×99 mm.

Fragment 6/9 is part of Bridget Riley's *Fragments* series, 1965, screen-printed on plexiglass, 625×720 mm, limited edition of 75 prints.

Valium is one of Damien Hirst's spot prints, 2000, lithographic print on gloss paper, 1200×1200 mm, limited edition of 500 prints.

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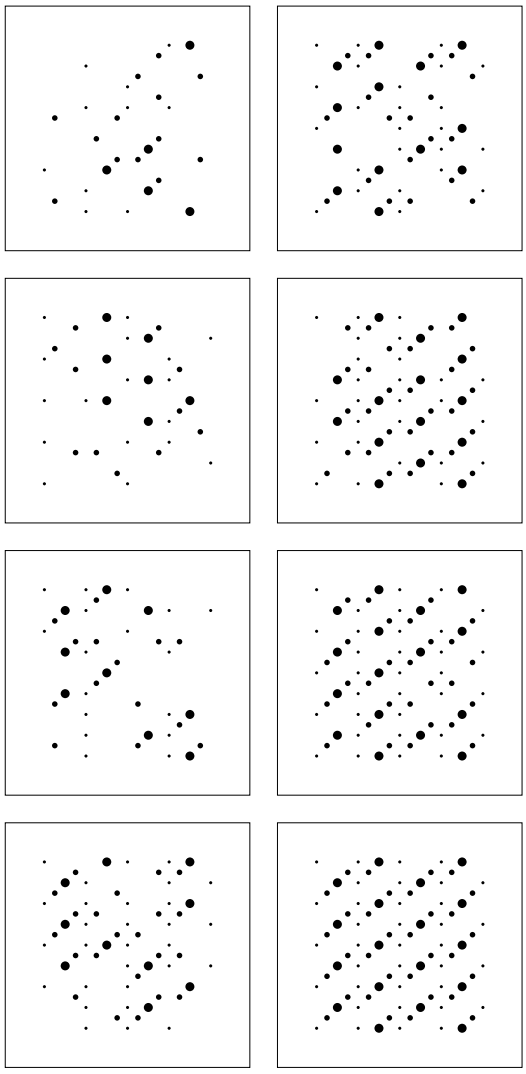

Figure 8: *Eight examples showing removal of (left column) 70%, 60%, 50%, 40%, (right column) 30%, 20%, 10%, 0% of the discs from the regular pattern in the bottom right, ©2008, Neil A. Dodgson, used with permission.*

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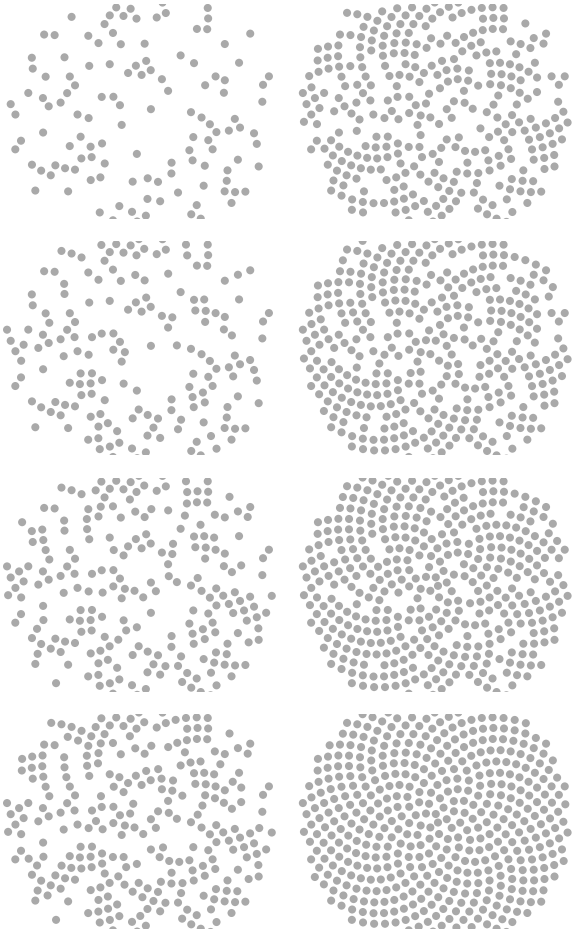

Figure 9: *Eight examples showing removal of (left column) 70%, 60%, 50%, 40%, (right column) 30%, 20%, 10%, 0% of the discs from the regular pattern in the bottom right. This pattern is inspired by Hirst's* Valium *(2000), ©2008, Peter Robinson, used with permission.*

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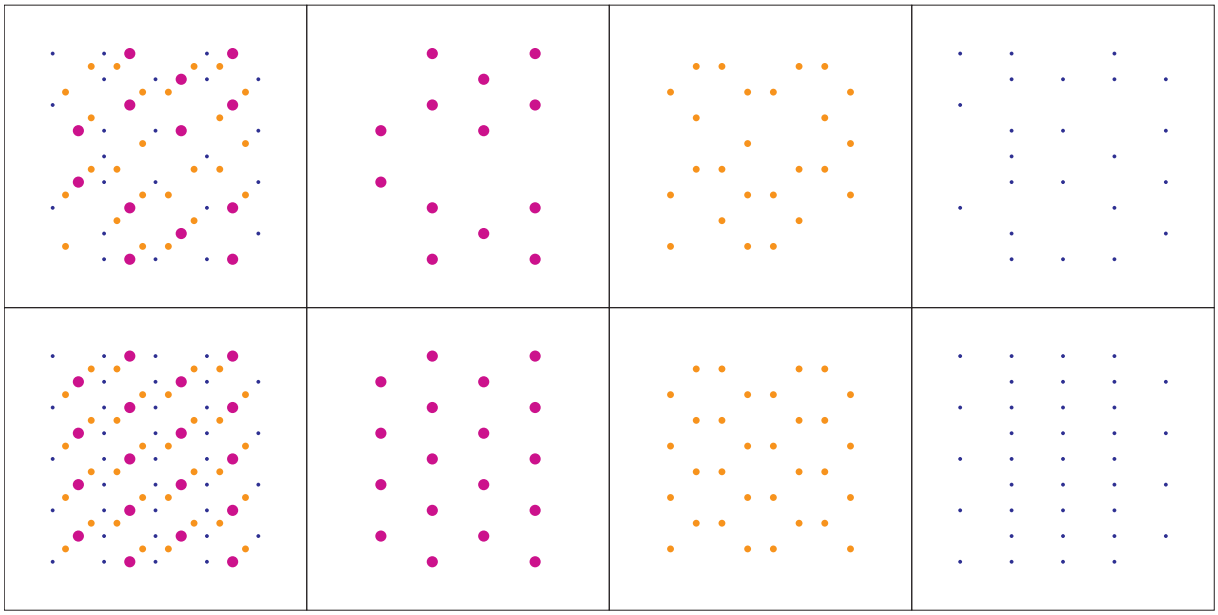

Figure 10: *A breakdown of* White Discs 2 *into the three component disc sizes. Top row: a computer generated, coloured version of the artwork. Bottom row: a coloured version of the underlying regular pattern.* ©*2008, Neil A. Dodgson, used with permission.*

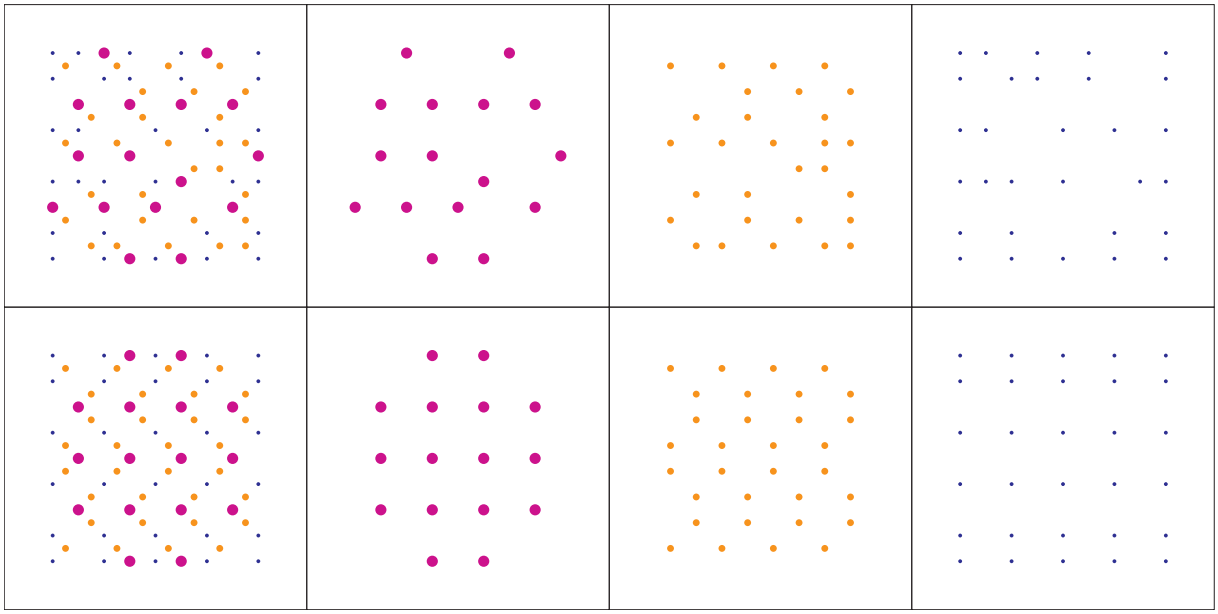

Figure 11: *A breakdown of* Fragment 6/9 *into the three component disc sizes. Top row: a computer generated, coloured version of the artwork. Bottom row: a coloured version of the putative underlying regular pattern.* ©*2003, 2008, Neil A. Dodgson, used with permission.*

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